# Mathematics | Calculus

| | Domain | Cluster | Standards |
|---|---|---|---|
| Functions, Graphs, and Limits | **Limits of Functions (F-LF)** | **Understand the concept of the limit of a function.** | 1. Calculate limits (including limits at infinity) using algebra.<br>2. Estimate limits of functions (including one-sided limits) from graphs or tables of data. Apply the definition of a limit to a variety of functions, including piece-wise functions.<br>3. Draw a sketch that illustrates the definition of the limit; develop multiple real world scenarios that illustrate the definition of the limit. |
| | **Behavior of Functions (F-BF)** | **Describe the asymptotic and unbounded behavior of functions.** | 1. Describe asymptotic behavior (analytically and graphically) in terms of infinite limits and limits at infinity.<br>2. Discuss the various types of end behavior of functions; identify prototypical functions for each type of end behavior. |
| | **Continuity (F-C)** | **Develop an understanding of continuity as a property of functions** | 1. Define continuity at a point using limits; define continuous functions.<br>2. Determine whether a given function is continuous at a specific point.<br>3. Determine and define different types of discontinuity (point, jump, infinite) in terms of limits.<br>4. Apply the Intermediate Value Theorem and Extreme Value Theorem to continuous functions. |

# Derivatives

| Domain | Cluster | Standards |
|---|---|---|
| Understand the Concept of the Derivative (D-CD) | Demonstrate an understanding of the derivative | 1. Represent and interpret the derivative of a function graphically, numerically, and analytically.<br>2. Interpret the derivative as an instantaneous rate of change.<br>3. Define the derivative as the limit of the difference quotient; illustrate with the sketch of a graph.<br>4. Demonstrate the relationship between differentiability and continuity. |
| | Understand the derivative at a point | 5. Interpret the derivative as the slope of a curve (which could be a line) at a point, including points at which there are vertical tangents and points at which there are no tangents (i.e., where a function is not locally linear).<br>6. Approximate both the instantaneous rate of change and the average rate of change given a graph or table of values.<br>7. Write the equation of the line tangent to a curve at a given point.<br>8. Apply the Mean Value Theorem.<br>9. Understand Rolle's Theorem as a special case of the Mean Value Theorem. |
| Computing and Applying Derivatives (D-AD) | Apply differentiation techniques | 1. Describe in detail how the basic derivative rules are used to differentiate a function; discuss the difference between using the limit definition of the derivative and using the derivative rules.<br>2. Calculate the derivative of basic functions (power, exponential, logarithmic, and trigonometric).<br>3. Calculate the derivatives of sums, products, and quotients of basic functions.<br>4. Apply the chain rule to find the derivative of a composite function.<br>5. Implicitly differentiate an equation in two or more variables.<br>6. Use implicit differentiation to find the derivative of the inverse of a function. |
| | Use first and second derivatives to analyze a function | 7. Relate the increasing and decreasing behavior of $f$ to the sign of $f'$ both analytically and graphically.<br>8. Use the first derivative to find extrema (local and global).<br>9. Analytically locate the intervals on which a function is increasing, decreasing or neither.<br>10. Relate the concavity of $f$ to the sign of $f''$ both analytically and graphically.<br>11. Use the second derivative to find points of inflection as points where concavity changes.<br>12. Analytically locate intervals on which a function is concave up, concave down or neither.<br>13. Relate corresponding characteristics of the graphs of $f$, $f'$, and $f''$.<br>14. Translate verbal descriptions into equations involving derivatives and vice versa. |
| | Apply derivatives to solve problems | 15. Model rates of change, including related rates problems. In each case, include a discussion of units.<br>16. Solve optimization problems to find a desired maximum or minimum value.<br>17. Use differentiation to solve problems involving velocity, speed, and acceleration.<br>18. Use tangent lines to approximate function values and changes in function values when inputs change (linearization). |

| | Domain | Cluster | Standards |
|---|---|---|---|
| Integrals | Understanding Integrals (I-UI) | Demonstrate understanding of a Definite Integral | 1. Define the definite integral as the limit of Riemann sums and as the net accumulation of change.<br>2. Correctly write a Riemann sum that represents the definition of a definite integral.<br>3. Use Riemann sums (left, right, and midpoint evaluation points) and trapezoid sums to approximate definite integrals of functions represented graphically, numerically, and by tables of values. |
| | | Understand and apply the fundamental Theorem of Calculus | 4. Recognize differentiation and antidifferentiation as inverse operations.<br>5. Evaluate definite integrals using the Fundamental Theorem of Calculus.<br>6. Use the Fundamental Theorem of Calculus to represent a particular antiderivative of a function and to understand when the antiderivative so represented is continuous and differentiable.<br>7. Apply basic properties of definite integrals (e.g. additive, constant multiple, translations). |
| | Calculate and Apply Integrals (I-AI) | Apply techniques of antidifferentiation | 1. Develop facility with finding antiderivatives that follow directly from derivatives of basic functions (power, exponential, logarithmic, and trigonometric).<br>2. Use substitution of variables to calculate antiderivatives (including changing limits for definite integrals).<br>3. Find specific antiderivatives using initial conditions. |
| | | Apply integrals to solve problems | 4. Use a definite integral to find the area of a region.<br>5. Use a definite integral to find the volume of a solid formed by rotating a region around a given axis.<br>6. Use integrals to solve a variety of problems (e.g., distance traveled by a particle along a line, exponential growth/decay). |